

A SURVEY OF PASSIVE MAGNETIC SPRINGS

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1. INTRODUCTION

This paper surveys methods for generating force using permanent magnets between two bodies over a range of displacements. For a range of exemplary configurations, we provide design guidelines and simple approximations useful for concept selection and design.

In the design of passive magnetic bearings, the goal is usually to maximize the (positive) stiffness over a small travel in the suspension direction while minimizing the destabilizing (negative) stiffness that arises in the transverse direction. Many configurations of permanent magnets with and without soft magnetic materials have been studied over the years [35], but the most basic building block is a pair of ring magnets set to attract or repel in the axial direction as sketched in Fig. 1(a).

At the other extreme, many applications require a low or negative stiffness over a large travel. For example, a constant-force generator can be used to decrease the gravity loads on an actuator for vertical motion [36]. A sketch of a simple “constant-force generator” is sketched in Fig. 1(e).

Between these extremes, many configurations have been proposed and studied in the literature. Li *et al* [18] optimize a gravity compensator based on three cylindrical permanent magnets as shown in Fig. 1(b). Janssen *et al* [12] study stacked arrays of magnets in bidirectional and Halbach arrangements. Zhang *et al* [46] optimize an arrangement as shown in Fig. 1(c) to minimize force variation over travel. Kimman *et al* [15] propose arrays of magnets arranged as in Fig. 1(d) to obtain a large force with low stiffness.

Analytical solutions useful for design are available for only the simplest configurations. A number of researchers have attempted to simplify equations or enhance expressions to aid in design [27]. Cal-

culations of forces between magnets can be calculated using equivalent charge models, equivalent coil models, and magnetic nodes (see for example [26, 2, 5, 14, 17, 8, 30, 31, 32, 33, 38, 25, 42, 48, 50]). For precise calculations, the permeability of the magnets must be added to the calculations [19, 45].

Magnet springs can be broadly separated into two groups, normal mode where the magnetization is in the same direction of the desired force and displacement (see for example [13, 29, 47, 44, 18, 43]) and shear mode where the force and displacement are orthogonal to the magnetization direction (see for example [15, 39, 9, 10, 7, 46]). Robertson [33] uses magnets inclined at an angle to get a desired force and stiffness. Berkhof [3] uses rotational angles to tune the spring. Torsion springs have also been made using magnets [23], [4], [16]. Zhou *et al* [49] and Mrak *et al* [24] tailor the geometry and magnetization to get a desired force-displacement relation. Configurations of magnet geometry and arrangements can be optimized, eg [21]. Halbach magnets can be used to improve force density, eg [7, 3, 22, 43].

Ferromagnetic material can be used to enhance the magnets but their presence makes hand calculations nearly impossible [11]. The presence of ferrous materials also leads to potential challenges of hysteresis and eddy current losses. Some mitigation approaches are possible. Pechacker *et al* used used pulses to tailor to magnetization [28] to eliminate power consumption. A magnet pulled into an iron tube (and variants of this topology) is a simple configuration that provides a near constant force over a relatively large range of travel and can be tailored for certain characteristics [37, 20, 34, 1, 3].

Earnshaw's theorem states that a configuration using only magnetic materials is unstable. Moser [22] showed that it can be stable if using diamagnetic

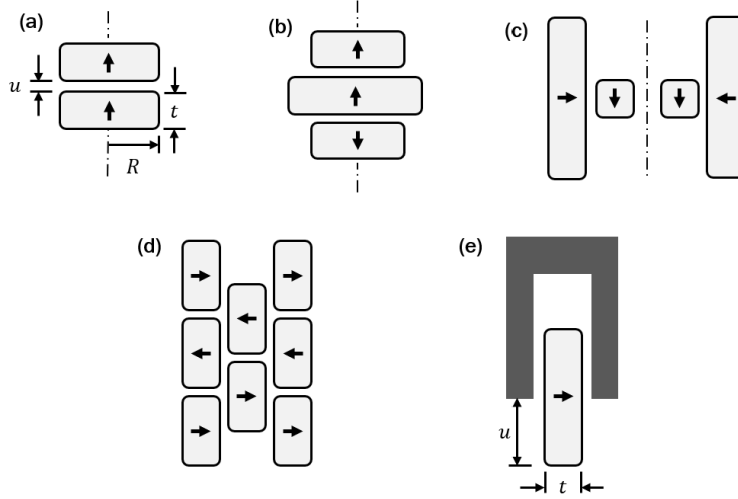


FIGURE 1. Sketches of various permanent magnet springs: (a) a pair of cylindrical magnets, (b) three cylindrical magnets arranged in a push-pull configuration [18], (c) a radially poled magnet arranged with an axially poled magnet [46], (d) an array of rectangular magnets forming opposing loops [15], and (e) a constant-force generator comprising a permanent magnet arranged within a soft-magnetic yoke [36].

materials. Earnshaw's theorem also states that the transverse and normal stiffness have a ratio of approximately 1:2 [IS IT APPROXIMATELY? OR EXACTLY? -1:2]. The negative stiffness of a magnet spring can be used to cancel positive stiffness for quasi-zero stiffness isolators [46, 41] or other applications [40].

Below we use simple models and finite-element analyses to develop some useful approximations and design heuristics for various layouts of magnets and iron in which the air gaps are sheared, with the goal of allowing a designer to quickly choose among concepts and perform rough sizing.

2. SHEAR-MODE MAGNET SPRINGS

A nearly constant force can be generated over a long travel using an arrangement as shown in Fig. 2.

Consider first the energy in a slice of thickness du within the tube, as shown in Fig. 2(c). If the magnet has remanent flux density B_r and we operate far from demagnetization, we can take $B = B_r + \mu_r \mu_0 H$. If the tube is sufficiently thick and composed of material of sufficiently high permeability, its reluctance is negligible compared to that within the magnet and air gap, and therefore the flux density within the magnet and air gap is approximately

$$B = B_r \frac{t/\mu_r}{t/\mu_r + 2g} \quad (1)$$

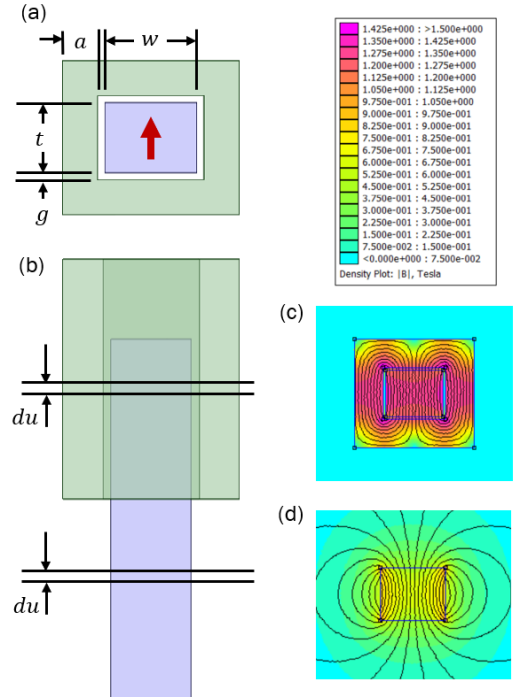


FIGURE 2. Bar magnet in a rectangular soft-magnetic tube: (a) top view, (b) side view, (c) magnetic flux density [T] at a section through the tube, and (d) magnetic flux density [T] at a section outside the tube. Flux density plots for N52 magnet in a 1018 iron tube with $t = 10$ mm, $w = 12.5$ mm, $g = 0.5$ mm, and $a = 6.3$ mm.

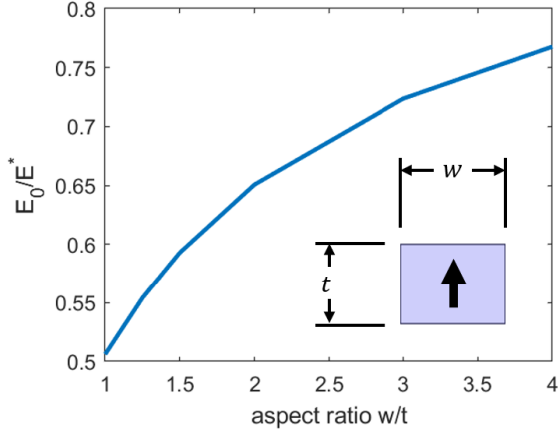


FIGURE 3. Nondimensional energy per unit volume in a rectangular magnet as a function of aspect ratio.

Defining the energy stored in the magnet as the work done on it relative to its state within a keeper ($H = 0$ and $B = B_r$), we can write the energy E_1 in the magnet and air gap as

$$\frac{E_1}{E^*} \approx \frac{2g}{t/\mu_r + 2g} \quad (2)$$

where $E^* du = (B_r^2 tw/2\mu_r\mu_0) du$ is the energy stored in the slice of magnet after magnetization. The flux density in the tube material is on the same order as that in the gap but its permeability is much larger, and therefore the energy in the tube material is negligible compared to that in the magnet and air gap.

Next consider a slice of magnet outside the tube as shown in Fig. 2(d). The flux density over the volume of the magnet and in the surrounding air cannot be simply approximated using magnetic circuits, but it can be obtained from 2D FEM or the demagnetization factor of the permanent magnet slice as studied by Brown [6] using the equation

$$\frac{E_0}{E^*} \approx \frac{1}{\pi} \left[2 \tan^{-1} \left(\frac{1}{p} \right) + p \ln(p) + \frac{1-p^2}{2p} \ln(1+p^2) \right] \quad (3)$$

where $p = t/w$. The relationship between E_0/E^* and aspect ratio is plotted in Fig. 3.

Equating the work done during a displacement du to the change in the energy stored in the magnet, we obtain an estimate of the force as

$$F \approx \frac{E_0 - E_1}{du} = \frac{B_r^2 tw}{2\mu_r\mu_0} \left(\frac{E_0}{E^*} - \frac{2g}{t/\mu_r + 2g} \right) \quad (4)$$

The upper bound on E_0 is $E^* = B_r^2 tw/2\mu_r\mu_0$, and the lower bound on E_1 is zero (e.g., with zero air

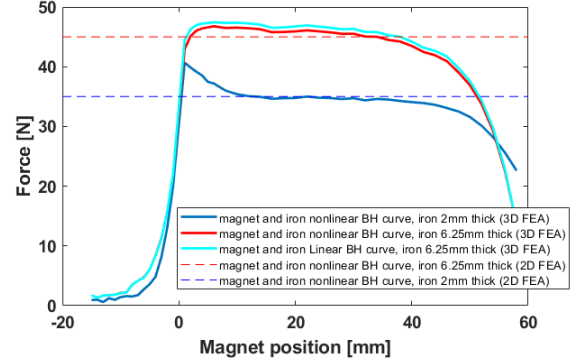


FIGURE 4. Force as a function of displacement for magnet in tube configuration from 3D FEM, with $t = 10$ mm, $w = 12.5$ mm, $g = 0.5$ mm, and the magnet length in the direction of travel 60 mm. The magnet begins to enter the tube when 'Magnet Position' is zero.

gap or reluctance in the tube). Therefore an upper bound on the force attainable from such an arrangement is the energy E^* stored in the magnet per unit length.

For the magnet as shown in Fig. 2(c), the aspect ratio $w/t = 1.25$, and from Fig. 3, $E_0/E^* \approx 0.55$. If the combined air gaps $2g$ are one tenth the magnet thickness, $E_1/E^* \approx 0.1$ and therefore the force attainable is approximately $0.45E^*$. For an N52 magnet with remanent flux density $B_r = 1.46$ T, relative permeability $\mu_r = 1.05$, and dimensions as given in Fig. 2, $E^* \approx 100$ J/m and therefore the predicted force F is about 45 N.

To attain this force, the reluctance of the iron tube must be low. In the example shown in Fig. 2(c), the tube is modeled as 1018 steel with wall thickness $a = 6.25$ mm (half the magnet width w). In this simulation the energy $E_1/E^* \approx 0.1$, attaining a value very close to that predicted by Eq. (2), and hence we expect this configuration to attain a force close to the predicted 45 N. Results of a three-dimensional finite-element model run for these dimensions with a magnet length of 60 mm are shown in Fig. 4, from which we see that...

If the iron is made substantially thinner than the magnet half width, considerable saturation occurs, causing more energy to be stored in the magnet and air gap, and thereby decreasing the predicted force. For example, if the wall thickness $a = 2$ mm, the energy ratio E_1/E^* doubles to approximately 0.2, and therefore we expect the force to decrease to $F \approx 35$ N. This result is in reasonable agreement

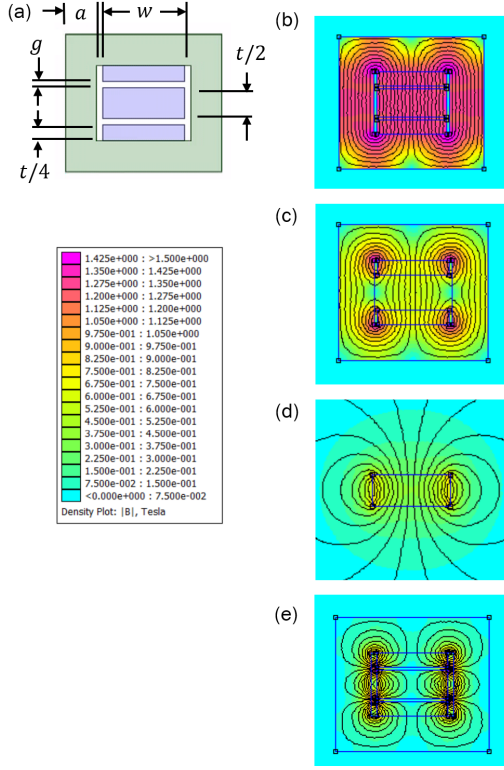


FIGURE 5. Bar magnet in rectangular tube with magnets fixed to its interior.

with the result shown in Fig. 4.

The foregoing analysis has assumed that the iron tube is not magnetized, but in practice hysteresis in the iron will cause it to retain some magnetization when the magnet is removed from a section of tube, so that the force on successive moves into and out of the tube can vary. This effect was measured in [JUSTIN INSERT CITE]. Unless strongly demagnetized, rare-earth magnets behave nearly linearly with low hysteresis [JUSTIN THERE'S A CITE FOR THIS?], and configurations in which the primary interactions is among magnets without soft magnetic materials are frequently analyzed in the literature [JUSTIN ALSO IS THERE A CITATION WE SHOULD USE HERE].

Next consider a configuration in which a moving magnet is centered in a soft magnetic tube with fixed magnets attached therein as shown in Fig. 5, and in which the fixed and moving magnets are poled in the same direction. In this case, the force is obtained from

$$F \approx \frac{E_0 + E_2 - E_1}{du} \quad (5)$$

where the E_0 is the energy stored in a slice with

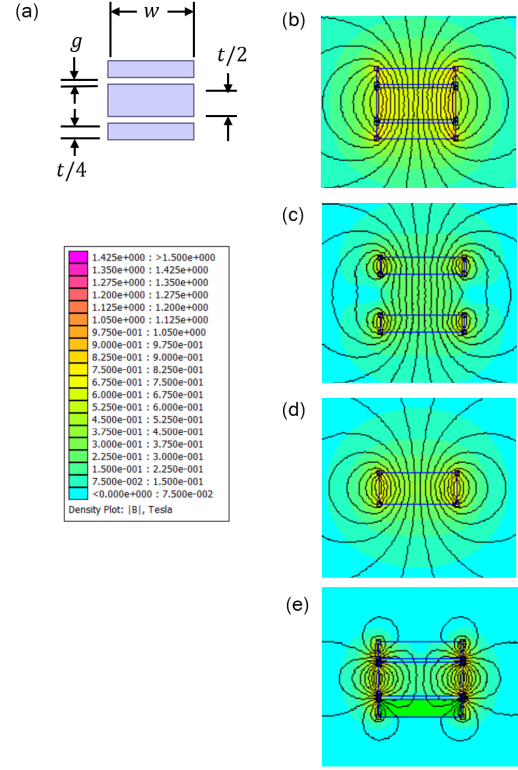


FIGURE 6. Cross sections of a slender bar magnet between two fixed slender bar magnets without a soft-magnetic tube.

magnet in air as shown in Fig. 5(d), E_1 is the energy stored in a slice with moving magnet inside the tube as shown in Fig. 5(c), and E_2 is the energy stored in a slice without the moving magnet as shown in Fig. 5(c). The energies E_1 and E_2 can be reasonably approximated by modification of Eq. (2), and the energy E_0 can be approximated from Fig. 3 to quickly obtain an estimate of the force for such a configuration. For the same total magnet thickness t , width w , and air gap g , the predicted force is slightly higher for this configuration than for the configuration without fixed magnets as shown in Fig. 2. The force as a function of displacement from 3D FEM for this case is also shown in Fig. 4.

If the moving and fixed magnets are poled in opposite directions, then the flux density in the magnets is driven low, and therefore E_1 in Eq. (5) becomes large than $E_0 + E_2$, indicating that the magnet is pushed out of the tube. Based on the simple magnetic circuit model as used in Eq. (2), we would expect E_1 to approach E^* , which for the dimensions given in Fig. 2, works out to $E^* \approx 100$ J/m, but the magnetic field in this case indicates significant fringing as shown in Fig. 5(e), from which we compute

that $E_1 \approx 82 \text{ J/m}$, and therefore we predict from Eq. (5) that $F \approx -20 \text{ N}$.

To eliminate the effects of hysteresis in the soft-magnetic tube, it is useful to consider a case where the fixed magnets are held in a non-ferrous tube or are otherwise supported. In this case, the flux plots corresponding to those of Fig. 5(b)-(d) appear as in Fig. 6. In this case the energies E_0 and E_1 can be approximated from Fig. 3, but we don't have a simple solution for E_2 , and therefore we obtain it from the 2D FEM. Because this configuration lacks a return path for the magnetic flux, the energy E_1 with magnet inserted into the tube is relatively large, and therefore the force for the dimensions of Fig. 2 using Eq. (5) are substantially smaller (about 16 N) than in the case including the soft-magnetic tube. If the polarity of the moving magnet is reversed, the flux density appears as in Fig. 6(e), from which we predict that the magnet is driven out from between the magnets by a force of about 14 N.

For moderate travels, a pair of moving magnets can be placed between fixed magnets with flux return paths provided by soft magnetic material that extends in the direction of travel as shown in Fig. 7(a). The magnetic circuit formed by this system is analogous to that of Fig. 5, and Eqs. (2) and (5) can be used to make course estimates of the force generated on each magnet. If the magnets extend much further into the page than their width, w/t is large and E_0/E^* approaches unity, but because the magnets do not extend far in the direction of travel, there is considerable fringing, and hence Eq. (5) somewhat over estimates the force. For large w/t a 2D FEM can be used to compute the forces as shown in Fig. 7(b) and plotted in Fig. 8. As the magnets are pulled downward from the centered position, we see that the force on each approaches $0.5E^*$, but they do not reach a nearly constant force regime before the force on the lower magnet M2 begins to drop and the force on the upper magnet M1 begins to increase as it is repelled by the lower pair of fixed magnets.

A configuration that has been widely studied is shown in Fig. 7(c) [JUSTIN PUT IN BEST CITES PLEASE]. In this case, we use only one moving magnet and extend it so that its bottom portion is repelled by the lower fixed pair. The combination of this repulsion with the attraction from the upper fixed pair leads to a relatively constant force near the center of travel as shown in Fig. 8 for magnet M3. Comparing Figs. 7(b) and (d), we note that

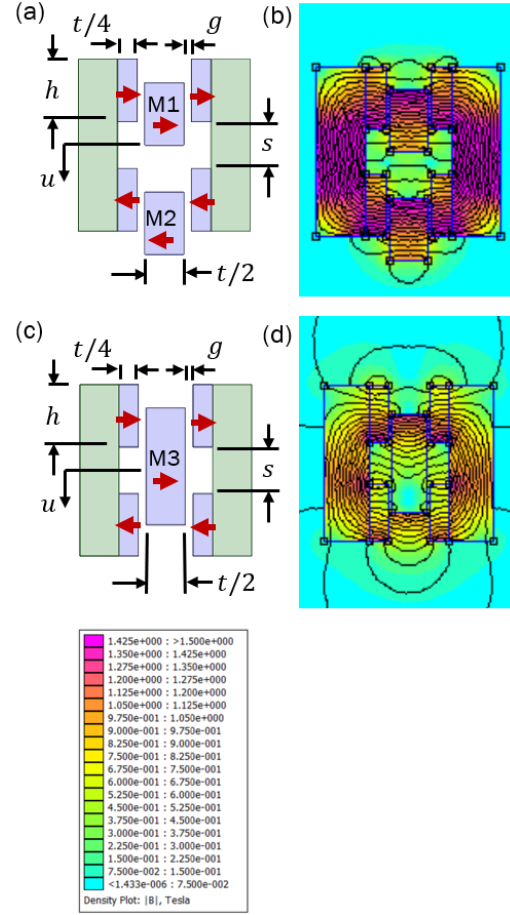


FIGURE 7. Arrangements of magnets and soft-magnetic materials with flux return paths in the direction of travel. Flux density plots for and N52 magnet, 1018 irons, $t = 10 \text{ mm}$, $g = 0.5 \text{ mm}$, $h = 8 \text{ mm}$, and $s = 6 \text{ mm}$.

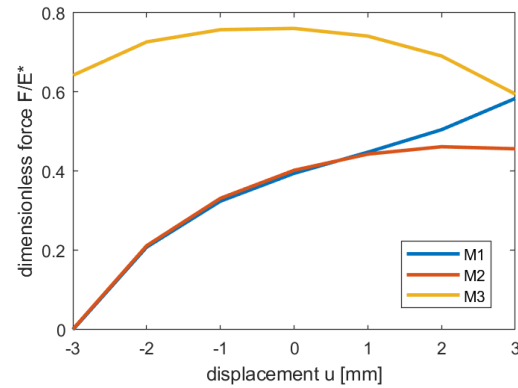


FIGURE 8. Nondimensional forces F/E^* from the 2D FEM for configurations with flux return paths in the direction of travel shown in Fig. 7.

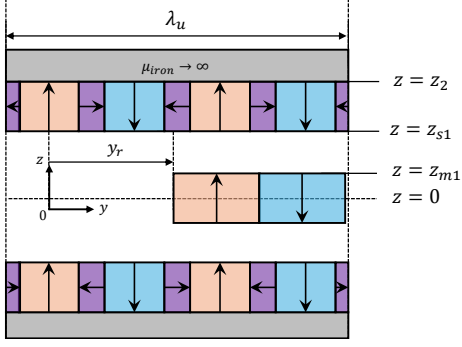


FIGURE 9. PM-PM configuration.

the flux density is substantially lower in this configuration than in the two-magnet configuration, and therefore the soft-magnetic return paths could be made thinner without substantially decreasing the force. Many researchers have studied such a configuration without soft-magnetic material [JUSTIN PUT IN BEST CITES PLEASE]

3. EXAMPLE – PM-PM CONFIGURATION

In this section, we calculate the magnetic force between permanent magnets (PMs) which is illustrated in Fig. 9. Depending on the working point in relative position y_r between Halbach arrayed fixed PMs and moving PMs, this can be used as either a gravity compensator or stiffness compensator. The fixed PMs are placed on the back iron to increase the magnetic field in the airgap. The permeability of the iron is assumed to be infinite. Additionally, we use the vacuum permeability μ_0 for the air and PM. We also assume that the analytical model is repeated with a spatial period of λ_u . We define the coordinates and boundaries where the material changes, as depicted in Fig. 9. Note that the model is symmetric about $z = 0$, and therefore we focus on the half of the model in vertical direction ($z > 0$) for calculation of the magnetic force.

Using the electric Lorentz force analogy, the magnetic force F_m is obtained by multiplying magnetic field intensity H and magnetic charge. The surface magnetic charge density σ_m replacing the moving PMs can be expressed in Fourier series as

$$\sigma_m(y, y_r) = \sum_{k=1}^{\infty} a_k(y_r) \cos pky + b_k(y_r) \sin pky, \quad (6)$$

where $a_k(y_r)$ and $b_k(y_r)$ are the Fourier coefficients for the k^{th} harmonic. We then calculate the magnetic horizontal force F_m using the y-component of the fixed PM-driven magnetic field H_{y1} as

$$F_m(y_r) = D_f \int_0^{\lambda_u} \sigma_m(y, y_r) H_{y1}(y, z_{m1}) dy \quad (7)$$

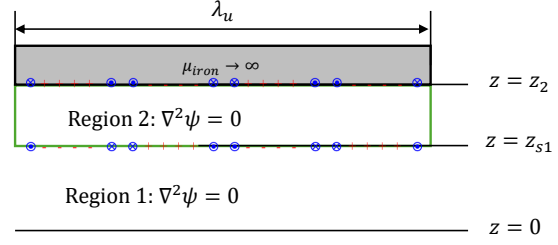


FIGURE 10. In the analytical model for field calculation, the fixed permanent magnets are represented by equivalent surface magnetic charge and current densities, allowing the use of the Laplace equation.

where D_f and λ_u are the motor depth and the horizontal length of the analytical model, respectively. The number in the subscripts in the field H represents the corresponding regions as depicted in Fig. 10. Note that σ_m is located at the upper surface of the moving PMs, so H_{y1} is evaluated at $z = z_{m1}$.

The fixed PMs (region 2) are represented as surface magnetic charge density σ_f and surface current density K_f for the vertically and horizontally magnetized PM, respectively. Since we have no volumetric magnetic sources in the system, the Laplace equation for the magnetic scalar potential ψ

$$\nabla^2 \psi = 0 \quad (8)$$

can be used, and the corresponding magnetic field H can be written as

$$H_{y1} = \sum_{k=1}^{\infty} -M_{1,k} \sinh(pkz) \cos(pky), \quad (9)$$

$$H_{z1} = \sum_{k=1}^{\infty} -M_{1,k} \cosh(pkz) \sin(pky),$$

$$H_{y2} = \sum_{k=1}^{\infty} \left(-M_{2,k} \sinh(pk(z - z_{s1})) - O_{2,k} \sinh(pk(z_2 - z)) \right) \cos(pky),$$

$$H_{z2} = \sum_{k=1}^{\infty} \left(-M_{2,k} \cosh(pk(z - z_{s1})) + O_{2,k} \cosh(pk(z_2 - z)) \right) \sin(pky), \quad (10)$$

where $M_{1,k}$, $M_{2,k}$, and $O_{2,k}$ represent the Fourier coefficients. In addition, p is the fundamental spatial harmonic whose period is λ_u . Note that the boundary condition in region 1 is applied to have no y-component at $z = 0$ due to the symmetry.

In order to find the Fourier coefficients in the fields, the surface charge and current densities should be represented in Fourier series as

$$\begin{aligned} K_f(y) &= \sum_{k=1}^{\infty} A_k \cos pky, \\ \sigma_f(y) &= \sum_{k=1}^{\infty} B_k \sin pky, \end{aligned} \quad (11)$$

where A_k and B_k are Fourier coefficients. The sign convention should be considered depending on the PM magnetization vectors at boundaries $z = z_{s1}$ and $z = z_2$, as illustrated in Fig. 10. The boundary conditions can be written as

$$H_{y2}|_{z=z_2} = -K_f \quad (12)$$

$$\begin{aligned} H_{z2}|_{z=z_{s1}} - H_{z1}|_{z=z_{s1}} &= -\sigma_f, \\ H_{y1}|_{z=z_{s1}} - H_{y2}|_{z=z_{s1}} &= +K_f, \end{aligned} \quad (13)$$

and therefore the Fourier coefficients for fields can be organized as

$$\begin{aligned} M_{2,k} &= \frac{A_k}{\sinh(pk\Delta z)}, \\ M_{1,k} &= \frac{1}{\sinh(pkz_2)} \times \\ &\quad \left[\sinh(pk\Delta z)(M_{2,k} + B_k) + \cosh(pk\Delta z)(-A_k) \right], \\ O_{2,k} &= \frac{1}{\sinh(pkz_2)} \times \\ &\quad \left[\sinh(pkz_{s1})(M_{2,k} + B_k) - \cosh(pkz_{s1})(-A_k) \right], \end{aligned} \quad (14)$$

where Δz is the height of the fixed PMs ($z_2 - z_{s1}$). By substituting (9) into (7), the magnetic force F_m can be expressed as

$$\begin{aligned} F_m(y_r) &= -\frac{\mu_0 D_f \lambda_u}{2} \times \\ &\quad \sum_{k=1}^{\infty} a_k(y_r) \frac{(1 - \cosh(pk\Delta z))A_k + \sinh(pk\Delta z)B_k}{\sinh(pkz_2)}. \end{aligned} \quad (15)$$

Note that $b_k(y_r)$ in (6) does not appear in F_m due to the orthogonality of the trigonometric functions. Furthermore, since (15) is the magnetic force for the half of the system ($z > 0$), the total force should be obtained by multiplying it by 2.

4. CONCLUSIONS

In precision machine applications, magnet springs find uses in vibration isolators and gravity compensators. Simple design formulas are not generally possible due to the behavior of magnetic fields as

well as the nonlinear BH curves of many magnetic materials. In this paper we survey the literature on the calculation of forces due to magnetic materials, and magnet spring design. We provide relatively simple methods for design of some magnet spring configurations.

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